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ORIGINAL PAPER

Robust adaptive sliding mode control for synchronization of space-clamped FitzHugh–Nagumo neurons

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Abstract Unlike taking the same external electrical stimulation to discuss chaotic synchronization in the literature, the synchronization between two uncouple FitzHugh–Nagumo (FHN) neurons with different ionic currents and external electrical stimulations is considered. The main contribution of this study is the application of a robust adaptive sliding-mode controller instead of the active elimination. The proposed sliding mode controller associated with time varying feedback gains cannot only tackle the system uncertainties and external disturbances, but also compensate for the mismatch nonlinear dynamics of synchronized error system without direct cancellation. Meanwhile, these feedback gains are not determined in advance but updated by the adaptive laws. Sufficient conditions to guarantee the stable synchronization are given in the sense of the Lyapunov stability theorem. In addition, numerical simulations are also performed to verify the effectiveness of presented scheme.

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1 Introduction

FitzHugh–Nagumo (FHN) neuron model was derived as simplified model of the Hodglein–Huxley (HH) neuron model [1] by FitzHugh [2] and Nagumo et al. [3]. In [4], the qualitative study of the FHN model was done and a variety of nonlinear phenomena were exhibited. As well as bifurcations of equilibria and limit cycles, a hard oscillation and separated loops can be demonstrated under suitable values of the system parameters. The quantitative study of chaos and bifurcation for FHN system was provided in [5] where applying the forward Euler scheme to discrete the differential equations.

Recently, to investigate processing of information in brain, the FHN neuron model is usually utilized to study neural firings due to the simplicity. Synchronization of chaotic neurons under external electric stimulation (EES) is attracted many interests during the last decade. With the development of control theory, various control schemes have been successfully applied to control and synchronization of chaotic neurons [6–9]. In [6], controlling chaos in FHN neuron by the adaptive passive method was introduced. Synchronization of two uncoupled FHN neurons in EES was found

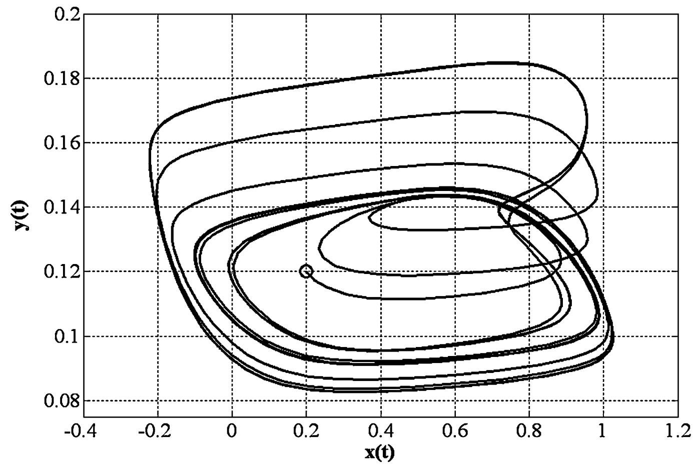

Fig. 1 State trajectory of the space-clamped HughFitzHugh–Nagumo neuron with parameter values $a = 0.25$, $b = 0.02$, $c = 0.005$, $I_1 = 0.082$, $I_m = 0.055$, $\omega_1 = 0.1$

in [7] where a nonlinear control technology was addressed, [8] where the author designed H-infinity variable universe adaptive fuzzy control, and [9] where a novel internal mode control method was proposed for the robust output synchronization. In [10], Hopf and Bogdanov–Takens bifurcations in a coupled FHN neural system with gap junction are investigated. Meanwhile, for two coupled FHN neurons in EES, feedback linearization control [11], back-steeping control laws [12, 13], robust adaptive sliding mode controls [14, 15], and robust adaptive fuzzy control [16] have been introduced to accomplish the chaotic synchronization.

The space-clamped FHN neuron subject to ESS is described by the following second-order nonautonomous differential equations [6]:

$$\begin{cases} \dot{x}_1 = -x_1^3 + (a+1)x_1^2 - ax_1 - y_1 + I_1 \\ \qquad + I_m \cos(\omega_1 t) \\ \dot{y}_1 = cx_1 - by_1 \end{cases} \tag{1}$$

where the first state variable, x_1, is the action potential, that is, the potential difference across the membrane. The second state variable, y_1, is the recovery variable which represents the effects of changes in those ion-channel conductances which tend to return the membrane potential to its resting level [5]; System parameters a, b, and c are positive constants; I_1 stands for the ionic current inside the cell and $I_m \cos(\omega_1 t)$ represents the ESS with the amplitude I_m and period $T_1 = 2\pi/\omega_1$. The dynamics of the space-clamped FHN neuron with EES was studied in [5, 17]. It is concluded that with the certain value of I_m the neuron experiences complicated chaotic firing. Meanwhile, the dynamic behavior of FHN neuron is very sensitive to the variation in the stimulation signal frequency. The space-clamped FHN neuron depicting chaotic dynamics for the parameter values sets, $a = 0.25$, $b = 0.02$, $c = 0.005$, $I_1 = 0.082$, $I_m = 0.055$, $\omega_1 = 0.1$ [5], with the initial conditions $(x_1(0), y_1(0)) = (0.2, 0.12)$, is as shown in Fig. 1.

In the past, the chaotic synchronization was discussed under the assumption of two FHN neurons had the same ionic current and EES [7–9]. In this study, the chaotic synchronized problem in the presence of system uncertainty and external disturbances between two uncoupled FHN neurons is considered with different ionic currents and external electrical stimulations. The introduced robust adaptive sliding mode control associated with time varying gains is shown to be able to compensate for the nonlinear dynamics of the synchronized error system without actively eliminating it as that commonly adopted in the traditional approaches [18–22]. Meanwhile, these time varying feedback gains are not to be determined in advance but updated adaptively. Based on the Lyapunov stability theory, sufficient conditions to guarantee the stable synchronization are given. In addition, numerical simulations are performed to show effectiveness of the presented scheme.

The rest of this paper is organized as follows. The formulation of synchronized problem and the controller design are addressed in Sect. 2. In Sect. 3, numerical simulations are performed to demonstrate the effectiveness of the proposed controller. In the final section, a concluding remark is made.

2 Problem formulation and control design

In the sequel, the synchronized problem between two uncoupled space-clamped FHN neuron with the case of different ionic currents and external electrical stimulations is considered. The master neuron is taken to be the form of (1). The slave neuron is chosen in the similar structure of (1) with system uncertainty, external disturbance, and can be described in the following:

$$\begin{cases} \dot{x}_2 = -x_2^3 + (a+1)x_2^2 - ax_2 - y_2 + I_2 \\ \qquad + I_s\cos(\omega_2 t) + \Delta(x_2, y_2) + d(t) + \phi(t) \\ \dot{y}_2 = cx_2 - by_2 \end{cases} \quad (2)$$

where x_2, y_2 are state variables of the slave system. I_2 is the ionic current inside the cell and $I_s\cos(\omega_2 t)$ represents the ESS with amplitude I_s and period $T_2 = 2\pi/\omega_2$. In this case, it is assumed that $I_2 \neq I_1$, $I_s \neq I_m$, $\omega_2 \neq \omega_1$. $\Delta(x_2, y_2)$ is the system uncertainty, $d(t)$ is the external disturbance, and $\phi(t)$ is the control to be determined. In general, the system uncertainty $\Delta(x_2, y_2)$ and the external disturbance $d(t)$ are assumed to be bounded as follows:

$$\begin{aligned} &0 \le |\Delta(x_2, y_2)| \le \Omega, \quad \forall x_2, y_2, \\ &0 \le |d(t)| \le D, \quad \forall t \end{aligned} \quad (3)$$

where Ω and D are positive constants. In addition, it is assumed that the slave neuron in (2) exists a unique solution in the time interval $[t_0, \infty), t_0 > 0$ for any given initial conditions. By taking account of the different ionic currents and the external electrical stimulations between systems (1) and (2), the synchronized problem is formulated in the following.

To proceed, the synchronized error states between systems (1) and (2) are defined as

$$e_x(t) = x_2(t) - x_1(t), \quad e_y(t) = y_2(t) - y_1(t). \quad (4)$$

Taking the time derivative of (4), the synchronized error system can be expressed as

$$\begin{cases} \dot{e}_x = F_1(x_1, x_2) + F_2(x_1, x_2) - ae_x - e_y \\ \qquad + (I_2 - I_1) + I_s\cos(\omega_2 t) - I_m\cos(\omega_1 t) \\ \qquad + \Delta(x_2, y_2) + d(t) + \phi(t) \\ \dot{e}_y = ce_x - be_y \end{cases} \quad (5)$$

where $F_1(x_1, x_2) = x_1^3 - x_2^3$ and $F_2(x_1, x_2) = (a+1)(x_2^2 - x_1^2)$ are nonlinear functions represented the mismatch dynamics. To this end, it is clear that the problem of synchronization is replaced by the equivalence of stabilizing the synchronized error system (5) by utilizing an appropriate control input $\phi(t)$. The goal of the current problem is to design the control $\phi(t)$ such that $\lim_{t\to\infty} e_x(t) \to 0$ and $\lim_{t\to\infty} e_y(t) \to 0$ for any initial conditions of the synchronized error system (5). It means that the behavior of the slave neuron can tend to that of the master neuron.

For chaotic synchronization of FHN neuron, the effects of nonlinear functions $F_1(x_1, x_2)$ and $F_2(x_1, x_2)$ are the key problem to be coped with. There were many approaches to compensate nonlinear effects of $F_1(x_1, x_2)$ and $F_2(x_1, x_2)$ in the literature, such as active control [7], backstepping methods [12, 13], the approximation theory [8, 14–16], and the adaptive sliding mode control associated with direct eliminations [20–22]. In this study, the robust adaptive sliding mode control with time varying feedback gains is introduced to tackle this problem.

Firstly, the synchronized error system (5) can be rewritten as follows:

$$\begin{cases} \dot{e}_x = [-f_1(x_1, x_2) + f_2(x_1, x_2)]e_x - ae_x - e_y \\ \qquad + (I_2 - I_1) + I_s\cos(\omega_2 t) - I_m\cos(\omega_1 t) \\ \qquad + \Delta(x_2, y_2) + d(t) + \phi(t) \\ \dot{e}_y = ce_x - be_y \end{cases} \quad (6)$$

where the functions $f_1(x_1, x_2) = x_1^2 + x_1x_2 + x_2^2$ and $f_2(x_1, x_2) = (a+1)(x_1 + x_2)$are bounded because of the bounded phase trajectories of $x_1(t), x_2(t)$.

The design approach of robust adaptive sliding mode controller involves two steps. (1) The appropriate sliding surface for desired sliding motion is selected. In the sliding surface, the slave neuron will be synchronous with the master neuron asymptotically. (2) The robust controller $\phi(t)$ is designed that brings any trajectory in phase space of the error dynamics to and stay in the sliding surface even in the events of system uncertainties $\Delta(x_2, y_2)$ and external disturbances $d(t)$.

Then, a sliding mode surface is chosen $\sigma(t) = 0$ [18], where

$$\sigma(t) = e_x(t) + ke_y(t), \quad k > 0 \quad (7)$$

From the second equation in (6), it obtains

$$e_x(t) = [\dot{e}_y(t) + be_y(t)]/c \quad (8)$$

Substituting (8) into (7) with $\sigma(t) = 0$ gives

$$\dot{e}_y(t) + (b + kc)e_y(t) = 0 \quad (9)$$

For $k > 0$, it is obviously that the stability on the sliding surface $\sigma(t) = 0$ is surely guaranteed. In the following, the control $\phi(t)$ of system (6) for achieving synchronization is proposed.

Theorem *If the control law $\phi(t)$ in system (6) is taken as follows*:

$$\phi(t) = e_y(t) - \left[K_0 + K_x(t)\left|e_x(t)\right| + K_y(t)\left|e_y(t)\right| + M\left|\sigma(t)\right|^N\right] \cdot \operatorname{sgn}\left(\sigma(t)\right) \quad (10)$$

where $\sigma(t)$ is the sliding surface defined in (7), $K_0 > |I_2 - I_1| + I_s + I_m + \Omega + D > 0$, $0 < N < 1$, and $M > 0$ are positive design constants and $\operatorname{sgn}(\bullet)$ *denotes the sign function, $K_x(t)$ and $K_y(t)$ are the adaptive feedback gains updated, respectively, according to the following adaptation algorithms*:

$$\dot{K}_x(t) = \rho_x\left|e_x(t)\right|\left|\sigma(t)\right|, \quad K_x(0) = 0,\ \rho_x > 0 \quad (11)$$

$$\dot{K}_y(t) = \rho_y\left|e_y(t)\right|\left|\sigma(t)\right|, \quad K_y(0) = 0,\ \rho_y > 0 \quad (12)$$

where ρ_x, ρ_y are the positive adaptation gains determining the adaptation process. The states of the synchronized error system (6) will asymptotically approach to and stay in the sliding surface $\sigma(t) = 0$.

Proof The Lyapunov function candidate of the problem is chosen as

$$V_1(t) = \frac{1}{2}\sigma^2(t) + \frac{1}{2\rho_x}\left(K_x(t) - \bar{K}_x\right)^2 + \frac{1}{2\rho_y}\left(K_y(t) - \bar{K}_y\right)^2 \quad (13)$$

where $\bar{K}_x, \bar{K}_y$ are positive constants and satisfy

$$\bar{K}_x > a + kc + |f_1| + |f_2| > 0, \quad \bar{K}_y > kb > 0 \quad (14)$$

Taking the time derivative of (13) along with the solutions of the synchronized error system (6), the selection of the sliding mode surface (7), and the controller (10), it yields

$$\begin{aligned}
\dot{V}_1 &= \sigma\dot{\sigma} + \frac{1}{\rho_x}(K_x - \bar{K}_x)\dot{K}_x + \frac{1}{\rho_y}(K_y - \bar{K}_y)\dot{K}_y \\
&= (-a - f_1 + f_2)e_x\sigma + \left[(I_2 - I_1) + I_s\cos\omega_2 t \right. \\
&\quad \left. - I_m\cos\omega_1 t + \Delta + d\right]\sigma \\
&\quad + kce_x\sigma - kbe_y\sigma - \left[K_0 + K_x|e_x| + K_y|e_y| \right. \\
&\quad \left. + M|\sigma|^N\right]|\sigma| \\
&\quad + (K_x - \bar{K}_x)|e_x||\sigma| + (K_y - \bar{K}_y)|e_y||\sigma| \\
&\le -\left[K_0 - \left(|I_2 - I_1| + I_s + I_m + \Omega + D\right)\right]|\sigma| \\
&\quad - (\bar{K}_y - kb)|e_y||\sigma| \\
&\quad - \left[\bar{K}_x - \left(a + kc + |f_1| + |f_2|\right)\right]|e_x||\sigma| \\
&\quad - M\left|\sigma(t)\right|^{N+1} \\
&< 0
\end{aligned} \quad (15)$$

From (15), since $V_1(t)$ is a positive definite and decreasing function, it follows that the zero equilibrium point ($\sigma = 0, K_x = \bar{K}_x, K_y = \bar{K}_y$) would be asymptotically stable. It means the states of the synchronized error system (6) will asymptotically approach to and stay in the sliding surface $\sigma(t) = 0$. On the sliding surface, the stability of synchronized error state $e_y(t)$ is surely guaranteed by choosing $k > 0$ according to (9) and induced the synchronized error state $e_x(t)$ approaching to zero. It follows that both of the synchronized error states will ultimately tend to zeros. As the control design meets the requirements depicted in this theorem, the synchronization between systems (1) and (2) is achieved. This completes the proof. □

The controller $\phi(t)$ can also be designed with constant feedback gains in the following form:

$$\phi(t) = e_y(t) - \left[K_0 + \bar{K}_x\left|e_x(t)\right| + \bar{K}_y\left|e_y(t)\right| + M\left|\sigma(t)\right|^N\right] \cdot \operatorname{sgn}\left(\sigma(t)\right) \quad (16)$$

By applying the controller (16) to the synchronized error system (6), the state trajectory in the phase space will approach to the sliding surface $\sigma(t) = 0$ in a finite time. For this case, the Lyapunov function of system (6) is selected as $V = \sigma^2$. By taking the time derivative of V, it yields the following inequality:

$$\dot{V} \le -M|\sigma|^{N+1} \le -MV^n, \quad 0 < n = \frac{N+1}{2} < 1 \quad (17)$$

which implies that $V^{1-n}(t) \le V^{1-n}(0) - M(1-n)t$, $t \in [0, t_s]$ and $V(t) = 0$ when $t \ge t_s > 0$ [23]. However, the controller proposed in (16) utilizes fixed feedback gains without depending on the values of initial synchronized error states. This means that the feedback gains seem to be large induced a kind of waste in practice. Thus, the controller in (10) associated with adaptive feedback gains significantly improves this drawback.

Remark 1 The controller in (10) demonstrates a discontinuous control law. To reduce the phenomenon of chattering, the sign function in the controller can be

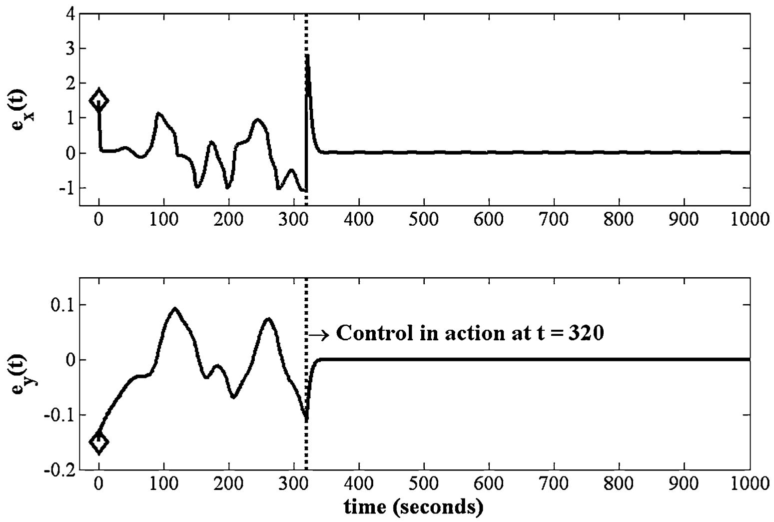

Fig. 2 Time responses of synchronized error states between two synchronized space-clamped FitzHugh–Nagumo neurons

modified as $\text{relay}(\sigma) = \sigma/(|\sigma| + \varepsilon)$ where ε is a sufficiently small design constant. With regard to the proof of stability for the modified sliding mode controller, one is referred to [24].

Remark 2 Controlling chaos in space-clamped FHN neuron by adaptive passive method was addressed by Wei et al. [6]. In fact, the robust adaptive sliding mode controller proposed in this study is also valid for the control of space-clamped FHN neuron with the presence of system uncertainties and external disturbances.

Remark 3 For chaotic synchronization, the various researches of adaptive sliding mode control schemes were addressed in the literature. The central idea of the past studies in [20–22] was applied the adaptive technique to estimate the switching gain of the compensation for system uncertainties and external disturbances. However, the mismatched nonlinear dynamics of synchronized error system were still to be cancelled directly by the equivalent control part of the sliding mode controllers in these researches. In the presence of system uncertainty and external disturbances, the main contribution of this study is the application of proposed robust adaptive sliding mode controller instead of the active elimination of nonlinear dynamics to achieve chaotic synchronization between two uncoupled FHN neurons with different ionic currents and external electrical stimulations.

3 Numerical simulations

In the sequel, the numerical simulation is performed to verify effectiveness of the proposed robust adaptive sliding mode controller. Using the fourth-order Runge–Kutta method with the initial conditions $(x_1(0), y_1(0)) = (-0.5, 0.75)$, $(x_2(0), y_2(0)) = (1.0, 0.6)$ and system parameters given in Fig. 1 to ensure the chaotic dynamics of the state variables, the synchronized error system (6) with the controller defined in (10) is numerically solved. The system uncertainty and the external disturbance are assumed to be $\Delta(x_2, y_2) = 0.15\sin(x_2)\cos(y_2)$ and $d = 0.15\sin(0.05\pi t)$, respectively. The ionic currents of two FHN neurons are selected as $I_1 = 0.082$ and $I_2 = 0.1$, respectively. The external electrical stimulations of two master and slave FHN neurons are assumed with different current amplitudes and frequencies. They are taken as $I_m = 0.055$, $\omega_1 = 0.1$, $I_s = 0.06$, $\omega_2 = 0.15$.

For the robust adaptive sliding mode controller described in (10) associated with (11) and (12), the positive design constants are chosen as $k = 45$, $K_0 = 2$, $M = 0.2$, $N = 1/2$, $\rho_x = 1$, and $\rho_y = 5$. In Fig. 2, it is shown that the synchronized error states oscillate irregularly when the controller is switched off, and when the controller is in action at $t = 320$ s, both of the synchronized error states converge to zero and the synchronization is achieved. Time responses of the sliding mode, the control signal, and the adaptive feedback gains are depicted in Figs. 3 and 4, respectively. It can be seen that the control signal is continuous and

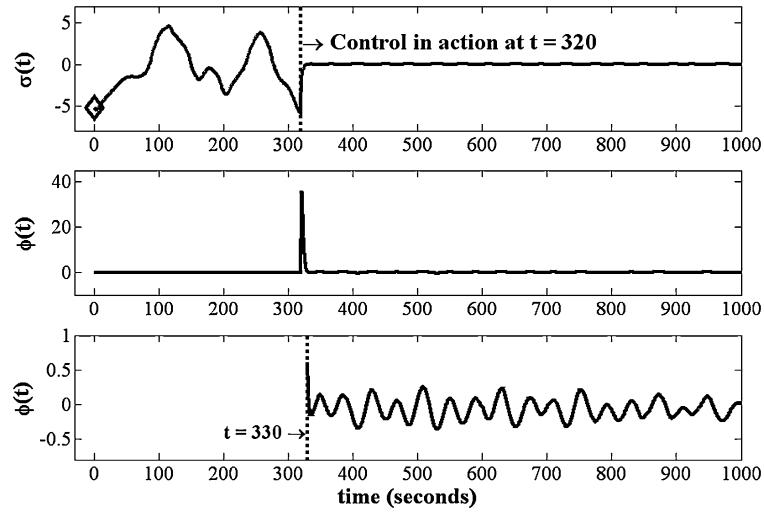

Fig. 3 Time responses of the sliding mode and control signal for slave space-clamped FitzHugh–Nagumo neuron

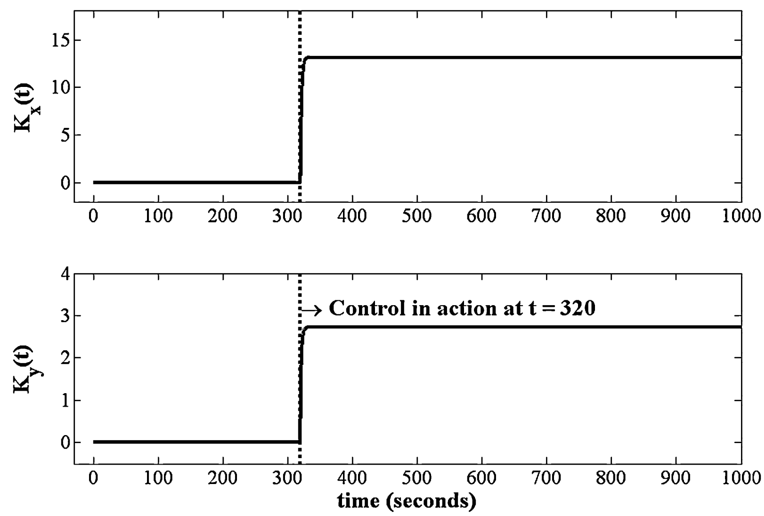

Fig. 4 Time responses of the adaptive feedback gains for slave space-clamped FitzHugh–Nagumo neuron

chattering free. In Fig. 5, time responses of state trajectories for the two synchronized space-clamped FHN neurons are illustrated. As expected, one can observe that the state trajectories of systems (1) and (2) separate from each other for different initial conditions. After the control in action at $t = 320$ s, all state variables tend to synchronize, where the slave neuron is with system uncertainties and external disturbances.

4 Conclusions

In this paper, by taking account of different ionic currents and external electrical stimulations, the robust adaptive sliding mode controllers has been addressed for achieving synchronization between two uncoupled space-clamped FitzHugh–Nagumo neurons with the presence of system uncertainties external disturbances. The designed controller has two adaptive feedback gains that can compensate the nonlinear dynamics without active elimination. In addition, these two adaptive feedback gains are not determined in advance, but updated according to the products of absolute values of sliding mode and synchronized error states. Based on the Lyapunov stability theorem, sufficient conditions guaranteeing synchronization are derived. Some numerical simulations are also performed to verify effectiveness of the presented scheme.

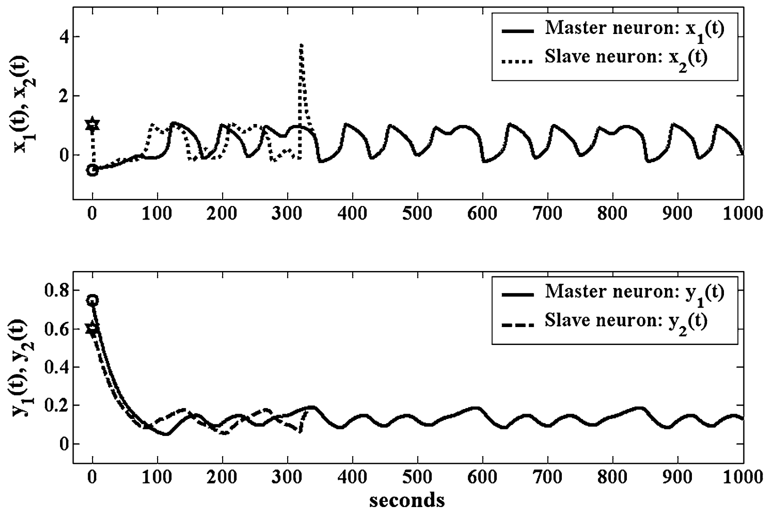

Fig. 5 Time responses of state trajectories for two synchronized space-clamped FitzHugh–Nagumo neurons

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